Polarization in multiparty systems

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Abstract
Polarization is widely seen to have important implications in politics and a large literature addresses the causes and consequences of polarization. It is, therefore, interesting that there is little consensus about how polarization should be measured and, sometimes, the measurement of appears to be a mere afterthought. While measuring polarization in two party systems is a relatively simple exercise, the same can not be said about multiparty systems. Here we aim to offer some insights into how we ought to go about measuring polarization by considering how polarization affects political outcomes in different political scenarios (e.g., legislative bargaining and coalition formation). We do not propose a ‘best’ measure of polarization but rather suggest that the choice of a measure should depend on the type of political problem under study. In other words, we demonstrate that it is essential that theoretical argument and measurement strategy are closely tied together.

Introduction
One of the key features of a political regime is the nature its party system. Political scientists characterize party systems along two main dimensions: i) their fragmentation and ii) their polarization. Fragmentation refers to the number and relative size of parties that inhabit the party system. Polarization refers generally to the ideological relationship that exists among parties, and more specifically to the ideological distance between parties and prevailing patterns of political coordination and competition among them (Bartolini and Mair 2007; Sartori 1976). Fragmentation and polarization are both posited to be correlates of regime performance and stability (Sani and Sartori 1983a, 600; Warwick 1994; Lijphart 1984; Powell 1982; Brown et al. 2011; Ostby 2008).

It is fair to say — despite some misgivings (e.g., Laver and Benoit 2015) and perhaps without sufficient critical examination — that the discipline has adopted Laakso and Taagepera’s (1979) index of the effective number of parties as the standard metric of fragmentation. In contrast, a variety of polarization measures are in circulation (e.g., Sigelman and Yough 1978a; Huber 1989a; Warwick 1994; Lupu 2015a; Dalton 2008; Alvarez and Nagler 2004; Dow 2011; Sani and Sartori 1983a; Reynal-Querol 2002) — no consensus exists on how to measure
polarization. Indeed, a surge of recent papers has debated how polarization ought to be conceptualized and measured (e.g., Stanig 2012; Maoz and Zeynep Somer-Topcu 2010; Esteban and Ray 1994a; Duclos, Esteban and Ray 2004; Best & Dow 2015; Schmidt (2017); Bauer 2017).

Our paper starts from the contention that many metrics of polarization — even many recently conceived measures — have paid insufficient attention to the dimensionality of the party system (Moaz 200x; Maoz and Zeynep Somer-Topcu 2010; Schmidt 2017). Most polarization measures assume a unidimensional party system, i.e., that it is sufficient to place parties along a single dimension. Moreover, even when scholars recognize the possibility that parties might compete in a multidimensional ideological space, they tend to adopt a dimension-by-dimension perspective on the problem (e.g., Alvarez and Nagler 2004; Dow 2011; Sani and Sartori 1983b; Esteban and Ray 1994a; Stanig 2012). What this unidimensional (or dimension-by-dimension) perspective misses (forgets is perhaps the better term) is that political competition takes on a fundamentally different character when it is conducted in a multidimensional ideological space than when it is conducted in a unidimensional policy space (McKelvey and Schofield 1986; Black 1948; Plott 1967). Of particular concern is that unidimensional politics is mainly unaffected by polarization in the sense that increasing or decreasing polarization in a one-dimensional party system has little impact on equilibrium outcomes or party strategies. In contrast, changes in polarization can have a significant impact on equilibrium outcomes or party strategies in a multidimensional party system. We illustrate our argument with a series of simple spatial models. We show in particular that the size and shape of the uncovered set is highly sensitive to “asymmetric polarization.”

**Polarization in Political Science**

One of the key challenges in measuring polarization is that it is not clear that there is a consensus among scholars about what ‘polarization’ is and, to a degree, the attitude towards polarization has been characterized by an attitude that we ‘know it when we see it’. In general terms, people generally take ‘polarization’ to mean that political parties are far removed from one another in ideological terms. Accordingly, political polarization takes a fairly simple form in two-party systems, i.e., the political parties are moving away from one another and taking increasingly distinct ideological positions that are perceived to make compromise difficult. This observation has led some scholars to focus on ideological distance in measuring polarization in multi-party systems; typically focusing on the ideological distance between the major parties on the left and the right or the most ‘extreme’ parties represented in the legislature.\(^1\)

Often, however, polarization is not merely seen as the result of political parties

\(^1\)For a few examples, see Huber (1989b), Crepaz (1990), Keman (1997), Mair and Castles (1997), Abedi (2002), Keefer and Stasavage (2003), Berglund et al. (2005), and Freire (2008), and Brown, Touchton and Whitford (2011) who all employ some variant of an ideological range or ideological distance measure for some or all of their analysis.
adopting more extreme positions but also the hollowing out of the ideological center. McCarty, Poole and Rosenthal (2016), e.g., note that while the political parties have pulled apart, moderates among members of Congress have gone missing. In this view, polarization is seen as a characteristic of the distribution of preferences or policy positions rather than a function of merely the extremes of the distribution’s support. A number of polarization measures have been proposed that take into account the distribution of party positions (as well as the parties’ sizes). Typically these are functions of measures of dispersion of the party positions (such as the variance or the standard deviation) as well as the parties’ sizes. The general form of these measures is \( \left( \sum \frac{v_i |(p_i - \hat{p})|^A}{B} \right)^C \) where \(A\) and \(B\) are normally constants (often with \(A \in \{1, 2\}\) and \(C \in \{5, 1\}\) while \(C\) is usually a constant but occasionally a function of the parties’ characteristics.\(^2\) For example, we can obtain Dalton’s measure of polarization by setting \(A = 2, B = .5,\) and \(C = 25: \sqrt[25]{\sum v_i (\frac{|p_i - \hat{p}|}{\bar{p}})^2} .\) More often than not, the scholars employing these measures appear more concerned with how parties spread out across the ideological spectrum than polarization.

A more explicit emphasis on the clustering of parties into separate and ideologically distant blocs appears, for example, in Esteban and Ray (1994\(^b\)) and Maoz and Somer-Topcu (2010). Again, these measures take into account the distribution of the parties in the ideological space and their sizes but are not directly based on familiar measures of dispersion. Esteban and Ray (1994\(^b\)), for example, are quite explicit about what they mean by ‘polarization’ and derive their measure from three axioms about how a polarization measure ought to respond to changes in the party system.\(^3\) Their proposed measure considers the distance between all pairs of parties: \( P_{ER} = K \sum_{i=1}^{n} \sum_{j=1}^{n} v_i^{1+\alpha} v_j |p_i - p_j| .\) While the measure, at first, looks quite different from those based on measures of dispersion from a central policy position, it is not fundamentally different. To see why, consider a simple measure of dispersion: \( \sum v_i |p_i - \hat{p}|\) where \(v_i\) is the vote share of party \(i\), \(p_i\) is the position of party \(i\), and \(\hat{p}\) is the weighted mean of the parties’ policy positions. Because \(\hat{p}\) is a function of all the parties’ policy position, the measure of dispersion is implicitly considering the weighted distances between any pair of parties.\(^5\) Thus, the measures have more in common then they appear to at first blush.

\(^2\)Examples of these include the measures used by Sigelman and Yough (1978\(^b\)); Dalton (2008); Lupu (2015\(^b\)), and Stanig (2012).

\(^3\)See Stanig (2012) for an example of the latter where \(C\) is the ratio of the actual to the effective number of parties.

\(^4\)It bears noting that Esteban and Ray (1994\(^b\)) are focused on societal polarization but there is a natural correspondence (better word?) between social groups and political parties in their framework.

\(^5\)To clarify, for party 1 in a three party system, \( |p_1 - \hat{p}|\) can be rewritten as \( |p_1 - (v_1 p_1 + v_2 p_2 + v_3 p_3)| = |(v_1 p_1 + v_2 p_2 + v_3 p_3) - (v_1 p_1 + v_2 p_2 + v_3 p_3)| = |v_2 p_2 - v_3 p_3| .\) Thus, in effect, the dispersion measure involves calculating the distances between each pair of parties but the difference is in how the differences in party positions are aggregated (i.e., note how the placement of the absolute values in the two measures implies that the differences in party positions partly, or fully, cancel out for centrist parties.

\(\sum\)
Another interesting aspect of polarization measures in the literature is that they, almost without exception, focus on polarization in single-dimensional issue spaces. An important exception here is Stanig (2012), who proposes a simple extension of his polarization measure based on city-block distances:

\[ P_{PS} = \frac{A}{N} \sum \frac{v_i |(p_{i1} - \hat{p}_1) + (p_{i2} - \hat{p}_2)|}{\sum v_i^2}. \]

Of course, as shown in figure 1, focusing on city block distances is not entirely satisfactory when it comes to measuring polarization in multi-dimensional spaces. The city-block distance in between \( P_1 \) and \( P_2 \) in the left panel of the figure is the same as the one in the right panel — while we would normally associate the party system in the right panel to be more polarized.

Noting this issue, Stanig (2012) proposes complementing the polarization measure with a measure of position congruence in order to distinguish between scenarios such as those depicted in figure 1. Other dispersion measures of polarization could, in principle, be extended in the same manner.

The focus on single-dimensional polarization measures is not unreasonable when studying two-party systems — even if the electoral competition takes place in multi-dimensional issue, the parties can be placed on a unique line (or in a single-dimensional subspace) going through the policy space. The reduction of the policy space to a single dimension in multi-party systems may not be such an innocuous assumption. A big chunk of the literature on party competition in multi-party systems is, e.g., explicit about the multi-dimensional nature of party competition. The literatures on issue ownership and valence competition are examples of where parties’ strategies involve taking positions on different issue dimensions. Thus, extensions such as the one proposed by Stanig (2012), are important when thinking about polarization in multi-party settings.

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6 For comparison, Stanig’s (2012) single-dimensional measure is \( P_{PS} = \frac{A}{N} \sum \frac{v_i |p_{i1} - \hat{p}|}{\sum v_i^2} \).

7 As this example shows, focusing on the Euclidean distance doesn’t solve the problem either as the Euclidean distance between the parties also happens to be the same here (although that it is not generally true).

8 See also Mogues (2008) for an extension of the Esteban and Ray (1994b) measure to two-dimensional spaces and Finseraas (2009) that adopts a sort of hierarchical approach to measuring polarization.
Our goal here is not to proclaim a particular measure of polarization as the most ‘appropriate’ measure of polarization or to propose a new and improved measure. Instead we aim to argue that there is often a mismatch between theoretical arguments focused on polarization and its measurement. This mismatch comes in two forms. The first comes from the colloquial use of the term polarization, i.e., instances in which scholars adopt polarization measures without careful consideration of how polarization is defined.

The second form of mismatch comes from a lack of specificity or detail in theoretical arguments. Theoretical arguments sometimes fail to specify how, or why, polarization affects the outcome of interest, i.e., a theoretical leap is made in asserting that polarization will have an effect. To clarify briefly what we mean here, consider a case of two parties bargaining over policy in a single dimension. Intuitively we might think that bargaining becomes more difficult when polarization increases. But we might also ask why that would be the case. Suppose we model the bargaining between the parties using a standard Rubinstein (1982) alternating offers bargaining model. In equilibrium in the Rubinstein (1982) bargaining model, the parties always reach an agreement and they always do so — irrespective of the degree of polarization (or, here, ideological differences). A critic of our approach might then argue that the invariance of bargaining success to polarization is a function of our decision to adopt such a simple and abstract model, which misses crucial aspects of what goes on in the bargaining. To which our response would be that this is exactly our point — our theory is no good at explaining how polarization matters. This, we claim, is not an uncommon problem with theories about polarization; theoretical arguments often rely, implicitly or explicitly, on spatial models of policy making that do not provide clear answer for why polarization should have an effect. Being more specific about those theoretical mechanism by which polarization matters is, obviously, of fundamental importance. Moreover, thinking hard about how polarization works can be helpful choosing an appropriate measure of polarization.

We proceed by considering series of simple spatial models that are commonly employed in the literature to analyze policy-making and demonstrate that the straightforward application of these models does generally not provide us with explanations of why polarization matters. Thus, if we believe these models describe some fundamental aspects of the policy-making process, then something is missing from our stories. We also aim to show that there is a particular class of models where change in polarization has the potential of affecting policy-making. In particular, we seek to demonstrate that polarization is more likely to

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9We have only touched on some the factors that differentiate the different polarization measures. Other factors that may be important in choosing a polarization measure may involve scaling the measures to allow for comparability across party systems or with the voting population.

10Admittedly, some of those instances occur simply because scholars mislabel (in our view) ideological differences as polarization. That is, the theoretically relevant construct is ideological differences and the measure is chosen appropriately but it is not labelled in the manner we would prefer.
affect policy-making when the policy space is multidimensional.\textsuperscript{11} This finding serves to emphasize the disconnect between theoretical development and the measurement of polarization as almost all the proposed measures of polarization focus on single-dimensional policy spaces.

\section*{Polarization \& Unidimensional Models of Politics}

In this section we examine how change in polarization affects policy-making or political outcomes in some familiar spatial models. In exploring these models, we consider the effects of changes in the party system (i.e., in the parties’ sizes and/or ideological positions). Rather than defining polarization precisely, we simply compare scenarios where we expect there to be a general agreement that the scenarios differ in terms of polarization. Thus, generally those scenarios reflect the two aspects of polarization that are common to most measures of polarization, i.e., a change in the ideological distance between ‘blocs’ of parties and/or hollowing out of the ideological center.

Polarization is sometimes seen to affect the outcome of legislative bargaining and being related to political stability, e.g., as a response to political gridlock (see, e.g., Sartori, 1976; Warwick, 1994). We begin by considering legislative bargaining under open rule and simple majority rule in a uni-dimensional policy space. Figure 1 depicts a legislative party system inhabited by five parties. Assume for the sake of simplicity that the parties are perfectly cohesive, that each possess equal voting weight, and that parties’ preference profile is single-peaked on the policy space $[0, 1]$. In Panel A the parties are uniformly distributed from 2 to 6. Panels B and C, in comparison, represent examples of more polarized legislatures. It is not obvious to us whether polarization is greater in panel B or panel C — that judgement ultimately depends on whether we attach greater weight to the presence of extreme ideological position or the hollowing out of the center.

Under an open agenda, the equilibrium policy outcome simple equals the median party’s position. The policy outcome will be 5 in panels A and C but 4 in panel B. Thus, if we wanted to attribute these changes to the effects of polarization, then those effects are rather limited. There are two things to note about the outcome of the legislative bargaining here. First, in comparing panels A and B, for example, on could argue that the change in policy outcome is the result of polarization. However, panel C makes it clear that increases in polarization do not necessarily affect policy outcomes. The relationship between polarization and policy outcome is, thus, largely an illusion, or, rather, it is a spurious relationship that comes from the fact that the degree of polarization and the location of the median voter will generally be correlated.\textsuperscript{12} It is, however,

\textsuperscript{11}Our use of ‘more likely’ here is decidedly ambiguous. To be slightly more precise, our focus is on the simple spatial models that are typically employed in the empirical literature, e.g., models that tend to focus on how procedures and institutions shape policy-making and incorporate few bells and whistles, e.g., uncertainty or incomplete information, that might result in polarization affecting the outcome.

\textsuperscript{12}This can easily be seen by noting that at extreme levels of polarization, when all the
clear that the theoretically relevant concept explaining policy outcomes is the latter — changes in the location of the median party affect policy outcomes while changes in polarization, as long as the median party stays in place, leave policy outcomes intact.

The second thing to note relates directly to the suggestion above that polarization risks political gridlock, policy immobilism, and, perhaps, political instability as a result. The model presented here does not offer grounds for such concerns; the outcome is always the median party’s preferred policy and the only circumstance in which there is no policy change is if status quo policy happens to be the median party’s preferred policy — i.e., in circumstances in which it is, perhaps, more difficult to complain about policy immobilism.\footnote{Once again, our claim is not that polarization does not matter in this regard but simply that our theoretical arguments occasionally make leaps of logic that are not warranted by the models’ assumptions. Applications of Tsebelis’s (2002) veto-player theory, including Tsebelis’ own\footnote{I think}, often argue that the size of the majority win-set is positively affects legislative productivity when it is not at all obvious why a larger win-set should have that effect; all that is required for a policy change is that the win-set is non-empty.}

Roemer & Rosenthal’s Setter Model

Romer and Rosenthal’s (1978) setter model resembles the legislative bargaining model except that it assumes that some legislator (or party) has agenda setting power and can present take-it-or-leave-it offers to the other parties in the legislature. As before, a proposal is adopted if a majority votes for it. In equilibrium, the setter (S) considers which policy proposals would gain favor with the median party and select the policy among those that it prefers the most. The policy outcomes will now depend on the preferences of the setter, the median voter, and the current policy (SQ) and equals arg\(\max_{p \in P} |p^S - p| \text{ s.t. } |p^M - p| < |p^M - SQ|\). Intuitively, party S offers its most preferred policy if party M also prefers it to M and otherwise it offers a policy that is the same distances from party M as SQ but is closer to party.
Figure 3: Roemer & Rosenthal’s Setter Model

In figure 3 we assume that the setter is the left-most party in the legislature and that the status quo policy is at 5.5. Much like in figure 2, polarization increases as we move from panel A to panel B. Yet, as the the change in the party system does not affect parties S and M, the equilibrium policy outcome equals 2.5 in both scenarios. Thus, increase in polarization does not necessarily affect policy outcomes.

In panel C, the distribution of the parties in the policy space is the same as in panel B but we now assume that identity of the setter party changes from the left-most party to second left-most party. Thus, the degree of polarization is the same in panel B and panel C. The equilibrium proposal is, however, now the setter party’s most preferred policy, i.e., 3. Thus, changes in polarization are neither a sufficient nor a necessary condition for changes in policy outcome. Again, this stands to reason as we know that the equilibrium outcome in the setter model is a function of the policy positions of the setter and the median voter. Any change in polarization that does not alter the $p_S$ or $p_M$ has not effect on the policy outcome.

Moreover, a change in $p_S$ or $p_M$ — even when those changes are interpreted as the result of increased polarization — is not guaranteed to affect the policy outcome. Any change in polarization where the $SQ$ lies between $p_S$ and $p_M$ before and after the change in the party systems does not affect the policy outcome.

In sum, much as in the open rule model of legislative bargaining, in empirical applications polarization may be correlated with the policy positions of the actors who do affect the outcome. However, as polarization is not guaranteed to affect the outcome, focusing on polarization instead of the theoretically relevant quantities ($p_S$, $p_M$, and $SQ$), opens up a risk of rejecting the hypothesis that polarization matters. Framed slightly differently, in the Romer and Rosenthal (1978) setter model, one should only expect polarization to matter some of the time.
Legislative Bargaining under Supermajority Rule

In some instances legislatures operate under supermajority rules rather than simple majority. Here we restrict our attention to what policy outcomes obtain under open rule (it is straightforward to examine a Romer and Rosenthal (1978) type model as well). As before we consider the situation of five parties of equal size but, rather than three parties being sufficient to accept a proposal, we now assume a supermajority of four parties is required. We refer to the parties by their rank from left to right, i.e., the left-most party is Party 1 and its policy position is $p_1$. The policy outcome under open rule is partly indeterminate, i.e., the precise outcome will depend on the order in which the parties are recognized to make a proposal. It is, however, clear that the policy outcome must be in the $[p_2, p_4]$ range, i.e., if a proposal that lies in that range has been adopted, then it is clear that there is no supermajority of four that would agree on adopting a policy that was further to the left or the right.

In panel A, then, the policy outcome will be in the $[4, 6]$ interval. In panel B, where there is greater polarization — at least according to some measures of polarization, the policy outcome will also be in the $[4, 6]$ interval. In panel C, where polarization is again greater than in panel A, the range in which the policy outcome will reside is larger, $[3, 7]$. Hence, greater polarization may affect the range of possible policy outcomes but it also may not. More specifically, the range only changes the policy positions of the pivotal supermajority parties, i.e., $p_2$ and/or $p_4$ change.

Bicameral Legislative Bargaining

Many legislatures are bicameral, requiring the assent of both chambers in order for a bill to be adopted into law. Under congruent bicameralism — where the two chambers resemble on another in their partisan composition — changes in polarization have much the same effect as it does in a unicameral legislature. We, therefore, confine our attention to incongruent bicameral legislature. We assume that the two left-most parties have greater representation in the upper chamber, leaving the second left-most party as the median in the upper chamber.
As in the unicameral case, the rules guiding the legislative bargaining matter from the outcome, i.e., whether the chambers operate under open or closed rule as well as the procedure guiding the ‘bargaining’ between the chambers. Regardless of the protocol, the equilibrium policy outcome will always lie in the interval \([p^{Mu}, p^{Ml}]\).\(^{14}\) Thus, the policy outcome is a function of the two legislators’ preferred policy positions (and the agenda setters’ if one exists) and the status quo. Thus, polarization does not affect the policy outcome in the game unless it affects the policy positions of at least one of these actors. For example, the policy outcome is not affected if the two right-most parties adopt more extreme policy positions (and, thereby, increasing polarization).

**Government vs. Opposition**

Dewan and Spirling (2011) advance a variation on Romer and Rosenthal’s (1978) agenda-setter model to explain the rigid pattern of government-vs-opposition voting in Westminster parliamentary systems. We explain Dewan and Spirling’s model with the aid of Figure 6 below, with 3 government members \((G_1^*, G_2, G_3)\) on the left of the policy space and the two opposition members \((O_1, O_2)\). Observe, however, that \(O_1\) lies to \(G_3\)’s left. Assume that as a government minister \(G_1^*\) can make a take-it-or-leave-it proposal to replace the status quo, \(SQ\), with some other policy, \(x\). Under sincere voting, the best outcome that \(G_1^*\) could secure would be \(x = O_1 - SQ\). Dewan and Spirling ask us to consider the outcome were \(O_1\) and \(O_2\) able to commit to vote against any ministerial proposal. If so, \(G_1^*\) could at best obtain \(x^* = G_3 - SQ\), an outcome that both opposition members prefer to \(O_1 - SQ\). The opposition members’ are thus able to credibly commit to vote en bloc against the minister’s proposals – and the government members’ best response is to vote en bloc for \(G_1^*\)’s proposal. The end result is that the opposition votes cohesively against the government and vice versa.

What happens to this government-opposition equilibrium when the situation in Panel A is polarized. One possibility (depicted in Panel B) is that the polarization in legislators’ ideal points leaves \(O_1\) to \(G_3\)’s left. These conditions leave the government-opposition equilibrium unchanged. Another possibility is that the polarization results in a complete separation of parties as in Panel C.

\(^{14}\)Obviously, this is not true where an agenda setter can make a take-it-or-leave proposals and the conditions for policy change are not met, i.e., when the agenda setter and one or both of the median legislator are on the opposite sides of the \(SQ\).
Under these conditions, it is possible that the opposition party would fail to vote cohesively. For example, if \( q \in [6, 7] \) and \( |x - O_1| < |x - O_2| \), then \( O_1 \) would vote for \( x \) whereas \( O_2 \) would oppose \( x \). But observe that this change in the outcome (i.e., a disunited rather than cohesive opposition) is not in fact due to increased polarization. For one thing, the opposition remains united in the face of increased polarization in Panel B. This shows that polarization is not sufficient to disrupt the equilibrium. Nor is polarization even necessary for the cohesive government-opposition voting pattern to fall apart; all that’s necessary for that to happen is that \( O_1 > G_3 \) such that there is no overlap between government and opposition parties. And this can happen without any increase in polarization whatsoever: simply interchange the positions of \( O_1 \) and \( G_3 \) in Panel A.

**Polarization in Multiple Dimensions**

As measures of polarization have focused on single issue dimensions, with a few notable exceptions, our notions of what polarization means when we move to multidimensional issue spaces are less developed. However, the key components that have been used to define polarization in single dimensional spaces have a fairly natural extension to multidimensional issue spaces. That is, we expect polarization to increase as the ideological distances between parties increase as well as when the parties cluster more close together to form blocs of parties. Thus, we distinguish between what we term symmetric polarization, which describes changes in the party system that are characterized by the parties moving further from one another ideologically, and asymmetric polarization, which describes changes that involve subsets of parties moving closer together. Figure 7 illustrates what we mean by symmetric and asymmetric polarization. The left panel of the figure shows symmetric polarization, where the parties

\[15\] However, it is not in equilibrium for \( G^*_1 \) to propose \( x \) such that \( |x - O_1| < |x - O_2| \) in response \( q \in [6, 7] \) unless one posits a model in which \( G^*_1 \) secures additional utility from dividing the opposition. Absent that additional assumption about \( G^*_1 \)’s utility, we would continue to observe that all opposition members cohesively oppose \( G^*_1 \)’s proposals whilst government members cohesively support them. Admittedly, the cohesion comes about simply because the two parties are internally homogeneous relative to the large ideological distance that separates them.
move away from the ideological center — in this particular case we have defined the center of the policy space as the centroid of the convex hull formed by the parties’ ideal points (i.e., the Pareto set). It is worth noting that symmetric polarization actually encapsulates both parts of the definition of polarization that we adopted in our discussion of polarization in unidimensional issue spaces. That is, here, symmetric polarization involves both an increase in the ideological distance between the parties and the hollowing out of the center. The right panel depicts asymmetric polarization. Here polarization increases as the result of parties $P_1$ and $P_2$ moving closer together, i.e., the party systems takes on a the structure of two blocs of parties. In this instance, the ideological distances between the parties do not change much (i.e., the distance of $P_3$ from the other parties) but yet the party system would be perceived as being more polarized.\textsuperscript{16}

We are interested in exploring how polarization affects policy outcomes when the policy space is multi-dimensional as well as whether the effects depend on whether it is symmetric or asymmetric polarization. In contrast with the simple spatial models considered above, equilibria are not guaranteed to exist when there are multiple issue dimensions. Or, more precisely, equilibria are almost guaranteed not to exist in the absence of institutions that structure the interactions between the parties. We adopt two approaches to examining the effects of polarization in multi-dimensional issue spaces. First, we consider a case of institutionally induced equilibrium by examining the effects of polarization in Laver and Shepsle’s (1996) model of coalition formation. Second, we consider the effects of polarization on the uncovered set.

Laver and Shepsle’s (1996) coalition formation model provides an elegant framework for analyzing coalition formation — although it has frequently criticized for relying on rather stringent assumptions. In their model, coalition bargaining revolves around the allocation of government portfolios to each of

\textsuperscript{16}We do not wish to overstate the differences between polarization in single and multiple dimensions. Symmetric polarization in unidimensional models simply means that all the parties move away from the center (but don’t necessarily become more similar in the process) while asymmetric polarization can be seen to describe a process where, say, the parties on the left become more ideologically similar.
the coalition parties where control of a government portfolio implies dictatorial powers within the portfolio’s remit. This, then, implies will always implement their preferred policy in the portfolios that they control and the policy space is effectively reduced to a lattice consisting of the parties’ ideal points across the dimensions (or portfolios). Figure 8 provides an example with three parties whose ideal policies are labelled $P_i$. We assume none of the parties controls a legislative majority. Under the portfolio dictator assumption, the only possible policy outcomes are, in addition to the parties’ ideal policies, represented by hollow circles. It is simple to verify that the equilibrium government will consist of either $P_1$ alone or a coalition of $P_1$ and $P_2$ (with $P_1$ setting the policy on the first dimension and $P_2$ on the second). We denote the equilibrium policy outcomes as $P_1P_1$ and $P_1P_2$.  

In figure 9 we examine whether polarization affects the equilibrium of the coalition formation game. The ‘new’ party positions are denoted $P'_i$. In the case of symmetric polarization, in the left panel of the figure, the equilibria remain the same in terms of party composition, i.e., $P'_1P'_1$ and $P'_1P'_2$ although, in terms of policy, the coalition policy now reflects the parties’ policy positions in the more polarized party system. Of course, the figure represents only one example of polarization, thus leaving open the possibility that a different degree of symmetric polarization might change the coalition composition. It bears noting, however, that $P'_1P'_2$ is the dimension-by-dimension median. $P'_1P'_2$ will, therefore, survive as an equilibrium as long as the polarization of the party system doesn’t change the identity of the median party on each policy dimension (i.e., it would require heterogeneity in the degree to which the parties adopt more extreme positions).

In the right panel of figure 9, asymmetric polarization is considered with parties $P_1$ and $P_3$ moving towards one another. Interestingly — even though

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17As Laver and Shepsle (1990) note, $P_1P_1$ is vulnerable to a majority in the legislature, i.e., a legislative majority prefers $P_1P_2$ to $P_1P_1$.

18In this particular example, the parties’ policy positions have been moved away from the centroid of the Pareto set by a third of the length of the median associated with the vertex associated with the party.
$P_1$ and $P_3$ are now more similar ideologically than before — the equilibrium coalitions remain the same as before; $P'_1P'_1$ and $P'_1P'_2$. Again, the main reason is that the identity of the dimension-by-dimension medians remains unaltered. In some ways it seems a bit ‘odd’ that increased polarization doesn’t render the $P_1$-$P_3$ coalition unstable but, on the other hand, both $P_1$ and $P_2$ are better off than they were in the less polarized party system.

As the identity of the dimension-by-dimension is of some relevance when it comes to determining the equilibrium outcome, we examine a couple of cases of asymmetric polarization in figure 10 where polarization takes by only one of parties $P_1$ and $P_2$ adopting a new policy platform. In both panels, party $P_2$ is no longer the median party on the second issue dimension. The equilibrium coalitions in the left panel are $P_1P_1$ and $P_3P_1$ (with $P_1P_1$ being preferred to $P_3P_1$ by a legislative majority). In this instance, a $P_1$ minority coalition might be considered the most likely outcome. Thus, polarization may not necessarily lead to the ‘majority bloc’ forming a coalition but may instead play into the hands of the more moderate parties within the bloc.

In the right panel, the equilibrium coalition is $P_1P_3$, i.e., we find that a within-bloc coalition — in line with what one might have expected intuitively.
The key difference between the two situations is not polarization (although they are not completely equivalent) but rather the fact that $P_1$ is the median party on both issue dimension in the left panel whereas in the right panel a different party is the median party on each of the dimensions.

**The Uncovered Set**

While decision majority rule works nicely in uni-dimensional issue spaces (provided the profile of preferences is single-peaked) and yields a stable equilibrium, those properties do not hold in more complex issue spaces. Indeed, equilibria are virtually guaranteed not to exist (Plott, 1967). The uncovered set has been proposed as a general solution concept and it has been showed that sophisticated voting outcomes will be members of the uncovered set (Miller, 1980; Shepsle and Weingast, 1984; Banks, 1985; McKelvey and Ordeshook, 1986). That is, there are good reasons to think that any policy outcome will be located in the uncovered set under majority rule. In informal terms, the uncovered set is the set of policies that are majority preferred to every other policy either directly or in two steps. That is, $x$ is in the uncovered set if for any other policy $y$ we either have $x \succ y$ or $x \succ z \succ y$ where $z$ is some other policy.

We explore how the uncovered set changes as symmetric and asymmetric polarization increases. We consider a party system consisting of five equal sized parties. The left-most panel in figure 11 is the ‘baseline scenario’. The middle panel considers symmetric polarization where each of the parties’ policy positions moves out from the centroid of the pentagon. As one might expect, the uncovered set expands but its shape remains much the same.\textsuperscript{19} The right-most panel considers asymmetric polarization — although it is asymmetric in a different manner from the examples above. That is, here we consider a scenario in which one of the parties moves away from the center of the policy space

\textsuperscript{19}There are some slight differences in the shape but they are minor and are most likely related to the algorithm used to compute the uncovered set.
while the rest of the parties’ policy positions remain the same. What we find is that asymmetric polarization does not have a big impact on the size of the uncovered set — it is slightly bigger than in the less polarized system and while the uncovered set expands slightly to the south-east, towards the extreme parties preferred policies, they do not expand far beyond the original uncovered set. Thus, changes in the position of a single party, even if fairly large, appear unlikely to have a significant impact on policy.

Conclusions

The literature on the causes and consequences of polarization has grown rapidly over the last few decades. Google scholar yielded 131 results for ‘political polarization’ in 1990. In 2016, the number of results had increased to 2,860. Political polarization. Of course, all those publication are not directly focused on the study of polarization but it is a fairly clear sign that polarization is attracting greater attention. It is, therefore, unfortunate that the discipline has not settled on a clear definition of polarization, i.e., sometimes it is merely taken to imply greater ideological differences while in other instances polarization is also seen to imply that absence of moderate political forces. Measurements of political polarization reflect this lack consensus.

While we consider there to value in clearly defining polarization and, subsequently, how it is measured, this is not our main concern in the current paper. Instead, we have sought to highlight that — when focusing on the polarization of party systems — polarization does not have direct implications for the equilibrium predictions of some our ‘workhorse’ models. Thus, we suggest that i) the emphasis on polarization in the study of party politics may be misguided, ii) that increases in polarization can derive from different changes in the party system, only some of which are relevant for affecting equilibrium outcomes, and iii) that the observed ‘effects of polarization’ may simply be due to the fact that polarization may be correlated with changes in theoretically relevant quantities (e.g., location of median legislator or agenda setter).

In sum, we believe that, as a general rule, we are better of thinking about the underlying theoretical processes and direct our attention on the factors that can be demonstrated to matter for the political outcome. In fact, we have a hard time thinking of (formal) models that focus explicitly on polarization. Polarization is a function of the choices made by individual political parties, i.e., it is a property of the party system rather than individual actors. Focusing on polarization is, therefore, likely to lead to less precise predictions about the outcomes of interest and, potentially, weaker results. This is not to say that polarization does not matter but its place within the analysis should perhaps (or sometimes) be slightly different, e.g., by considering how changes in theoretically relevant factors (such as the position of the median legislator) are related to the process of polarization.

We plan to examine asymmetric polarization as defined above at a later point.
In this paper we have only considered a few models of party politics in order to demonstrate how increases in polarization do not necessarily have any implications — and, admittedly, the models considered here might be considered straw men of sorts. As we move forward we are interested in examining a wider range of models of interest to scholars of party politics and, in particular, more model of multi-dimensional policy spaces. This, of course, is particularly important as most multi-party systems are characterized by competition along multiple issue dimensions. As the interest in polarization spreads to multi-party systems, it also becomes increasingly important to think about both polarization ‘means’ in multi-dimensional systems and also how to measure it.

References


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